MTL 106 (Introduction to Probability and Stochastic Processes)

II Semester 2016-17

Tutorial Sheet 4

MGF, Characteristic Function and Expectation

1. Prove that for any random variable X, E[] ≥ . Discuss the nature of X when one have equality.
2. MGF of a random variable Y is given by MY(t) = Find the pmf/pdf of the random variable
3. State True or False:
   1. If the characteristic function of a random variable W is CW(t) = , then P(1 < W ≤ 5) = .
   2. Let X be a discrete random variable with taking values , k = 0, 1, 2, … and such that P(X =) = , Var(X) exists.
4. Let Φ be the characteristic function of a random variable X. Prove that 1 - | Φ(2u) |2 ≤ 4(1 - | Φ(u) |2)
5. Consider the random variable X with E(X) = 1 and E(X2) = 1.
   1. Find E[(X – E(X))4] if it exists.
   2. Find P( < X ≤ 3) and P(X = 0)
6. The mgf of a random variable is given by MX(t) = exp(µ())
   1. What is the distribution of X
   2. Find P(µ - 2σ < X < µ + 2σ), given µ = 4
7. Let X be a random variable with Poisson distribution with parameter λ. Show that the characteristic function of X is exp[λ()]. Hence, compute E(X2), Var(X) and E(X3)
8. Let X be a random variable with N(0, σ2). Find the moment generating function for the random variable X. Deduce the moments of order n about zero for random variable X from the above result.
9. The moment generating function of a discrete random variable X is given by MX(t) = . If µ is the mean and σ2 is the variance of this random variable, find P(µ - σ < X < µ + σ).